MA181 - Week 2 Solutions

Lecture 1 Problems

Question 1

$A = \{-3, -1, 0, 1, 2, 5\}$ and,

$B = \{-1, 3, \pi, 5\}$ so:-

(i) $A \cup B = \{-3, -1, 0, 1, 2, 3, \pi, 5\}$

(ii) $A \cap B = \{-1, 5\}$

(iii) $A \setminus B = \{-3, 0, 1, 2\}$

(iv) $A \triangle B = \{-3, 0, 1, 2, 3, \pi\}$

(v) $|A| = 6$

(vi) $|B| = 4$

(vii) $|A \times B| = 24$

(viii) $A \times B = \{(-3,-1), (-3,3), (-3,\pi), (-3,5),$
$(-1,-1), (-1,3), (-1,\pi), (-1,5),$
$(0,-1), (0,3), (0,\pi), (0,5),$
$(1,-1), (1,3), (1,\pi), (1,5),$
$(2,-1), (2,3), (2,\pi), (2,5),$
$(5,-1), (5,3), (5,\pi), (5,5)\}$

Question 2

$\mathcal{U} = \{x^2 : x \in \mathbb{N} \text{ and } x < 7\}$

$C = \{4, 9, 16\}$

(i) $\mathcal{U} = \{1, 4, 9, 16, 25, 36, 49\}$

(ii) $\overline{C} = \{1, 25, 36, 49\}$

(iii) $|\mathcal{U}| = 7$

(iv) $|\overline{C}| = 4$

(v) $C \times C = \{(1,1), (1,25), (1,36), (1,49),$
$(25,1), (25,25), (25,36), (25,49),$
$(36,1), (36,25), (36,36), (36,49),$
$(49,1), (49,25), (49,36), (49,49)\}$

Question 3

(i) $A \cap B = A \iff A \subseteq B$

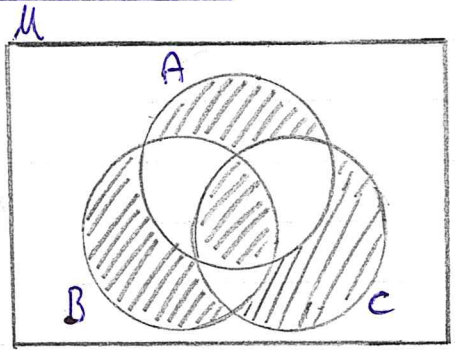
(ii) $A \cup B = A \iff B \subseteq A$

(iii) $\overline{A} \cap \mathcal{U} = \emptyset \iff \mathcal{U} = \emptyset$

(iv) $\overline{A \cap B} = \overline{B} \iff \overline{\overline{A \cap B}} = \overline{\overline{B}} \iff A \cap B = B \iff B \subseteq A.$

Lecture 2 Problems

Question 4



We can see that the symmetric difference is associative, i.e.

$$A \Delta (B \Delta C) = (A \Delta B) \Delta C$$

as $x \in U$ is in $A \Delta (B \Delta C) = (A \Delta B) \Delta C$ iff it is in exactly one of the three sets A, B, C or it is in all of them.

Question 5

$$(A^c \cap B)^c \cap (A \cup B) = ((A^c)^c \cup B^c) \cap (A \cup B) \quad \text{by 10b}$$

$$= (A \cup B^c) \cap (A \cup B) \quad \text{by 7}$$

$$= A \cup (B^c \cap B) \quad \text{by 4a}$$

$$= A \cup (B \cap B^c) \quad \text{by 3b}$$

$$= A \cup \emptyset \quad \text{by 8b}$$

$$= A \quad \text{by 5a}$$

Dual: $(A^c \cup B)^c \cup (A \cap B) = A$

Week 2 Homework Questions

Question 6

$M = \{1, 2, 3, \ldots, 10\}$

$A = \{1, 4, 7, 10\}$

$B = \{1, 2, 3, 4, 5\}$

$C = \{2, 4, 6, 8\}$

(i) $A \cup B = \{1, 2, 3, 4, 5, 7, 10\}$ (5)

(ii) $A \cap C = \{4\}$ (5)

(iii) $A - B = \{7, 10\}$ (5)

(iv) $\overline{M} = \phi$ (5)

(v) $(A \cup B) - (C - B) = \{1, 2, 3, 4, 5, 7, 10\} - \{6, 8\}$

$= \{1, 2, 3, 4, 5, 7, 10\}$ (5)

25 marks

Question 7

(i) $\{x\} \subset \{x\}$ False (2)

The sets are equal so the inclusion is not proper. (3)

(ii) $\{x\} \in \{x, \{x\}\}$ True (5)

(iii) $\{x\} \in \{x\}$ False (2)

$\{x\}$ is not a member of the set containing only x. (3)

(iv) $\{x\} \subseteq \{x, \{x\}\}$ True (5)

(v) $A \times B = B \times A$ False (2)

Counterexample: $A = \{1\}$, $B = \{2\}$

$$A \times B = \{(1,2)\} \neq \{(2,1)\} = B \times A$$ (3)

(vi) $\overline{A - B} = \overline{B - A}$ False (2)

Counterexample: Take A, B as in part (v).

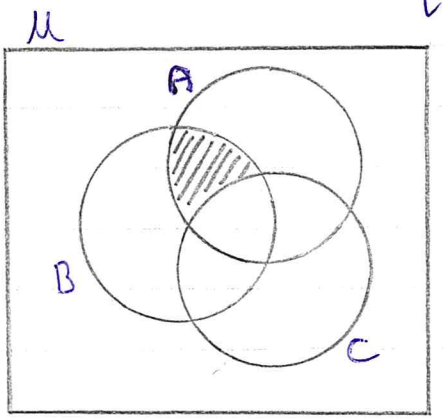
$\mathcal{U} = A \cup B = \{1, 2\}$, so

$$\overline{A-B} = \overline{\{1\}} = \{2\} \neq \{1\} = \overline{\{2\}} = \overline{B-A}$$ (3)

(vii) $A \cap (B \setminus C) = (A \cap B) - (A \cap C)$ True (5)

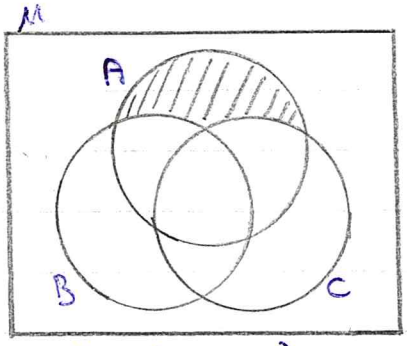
Part (vii) justification :-

Both sides are equivalent to $A \cap B \cap \overline{C}$ i.e.

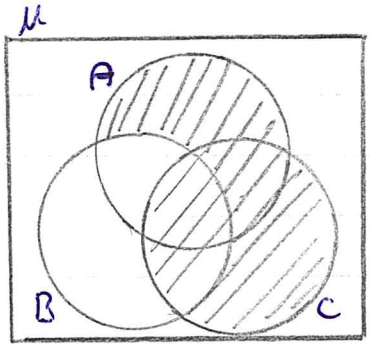


(viii) $P(\phi) = \{\phi\}$ True (by definition) (5)

(ix) $A - (B \cup C) = (A - B) \cup C$ False (2)



$A - (B \cup C)$



$(A - B) \cup C$ (3)

45 marks

Question 8

$(\mathcal{U} \cap A) \cup (B \cap A) = (A \cap \mathcal{U}) \cup (A \cap B)$ by 3b (twice)

$= A \cap (\mathcal{U} \cup B)$ by 4b

$= A \cap (B \cup \mathcal{U})$ by 3a

$= A \cap \mathcal{U}$ by 6a

$= A$ by 5b

(10)

Dual: $(\phi \cup A) \cap (B \cup A) = A$ (5)

15 marks

Question 9

(i) $A \times B = \{1, a, x\} \times \{1, b\}$

$= \{(1,1), (1,b), (a,1), (a,b), (x,1), (x,b)\}$ (5)

(ii) $B \times A = \{1, b\} \times \{1, a, x\}$

$= \{(1,1), (1,a), (1,x), (b,1), (b,a), (b,x)\}$ (5)

(iii) $(A \times B) - (B \times A) = (i) - (ii)$

$= \{(1,b), (a,1), (a,b), (x,1), (x,b)\}$ (5)

15 marks