Lab A on week 2: Complex numbers and rotation

Activity 1

Type the following code into Matlab (note that here and everywhere below you may copy and paste instead of typing).

```
a = 2 + 3i
```

The variable a represents a point on the plane with coordinates 2 and 3; that is, the number 2 standing on its own represents the horizontal coordinate, and the number 3 standing with i represents the vertical coordinate. To check this, let us plot this point on the plane. Use the following command.

```
plot(a)
```

The command above opens a new window with a plot, but we cannot see the point a on this plot. This is because, by default, the plot command plots lines connecting points, but not the points. If we want to see the point a, we need to tell Matlab explicitly that it must show points. One way of plotting the point a so that we can see it is the following command; please use it.

```
plot(a, '*')
```

The command above plots an asterisk at the position of the point a. Now we can see it. There are other options for plotting points (and lines); you can see them all by typing the following command. Please try it; it gives you lots of information about plotting graphs in Matlab, but perhaps the most interesting part is how you can change the colour of the graph and the style of points and lines.

```
help plot
```

Of course, you may plot more than one point on one graph. Try the following commands:

```
a = 1+i
b = 1-i
plot([a, b])
```

The commands above draw a line from the point a with coordinates (1,1) to the point b with coordinates (1,-1).

Activity 2

A point written as a number plus a number with i, such as 2 + 3i, is called a complex number. Apart from its horizontal coordinate and its vertical coordinate, two more numbers are associated with a point written as a complex number:

- the distance from the origin to the point, and

- the angle between the positive direction of the horizontal axis and the direction from the origin to the point

Use the following commands to find the distance and the angle for the point a:

```
abs(a)
angle(a)
```

The angle is expressed in radians; this is convenient in some situations and inconvenient in others. To convert the angle from radians to degrees, use the following command. Note that here we use a special word ans; in Matlab it always means the answer produced by the previous command.

```
rad2deg(ans)
```

Now please use the same commands to find the distance and the angle for the point b; before you find them using Matlab, try to guess what their values are.

Activity 3

Now let us experiment with multiplication of complex numbers. Multiplying by a complex number is always interpreted as rotating anti-clockwise by the angle corresponding to that number. Try these commands:

```
angle(i)
rad2deg(ans)
```

The answer produced by the commands above shows you what to expect when you multiply by i. Let us try rotating the plane by multiplying points on the plane by i. Use the following commands.

```
a = 1
b = i
c = -1
plot([a, b, c])
```

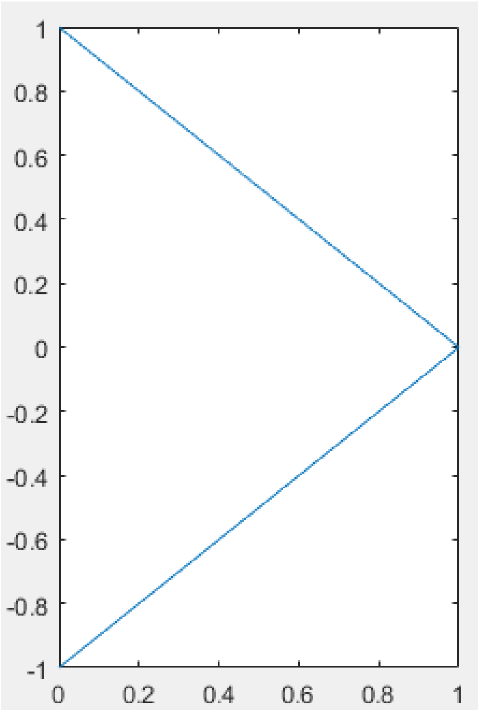
The commands above produce a simple roof-shaped figure. We can rotate it anti-clockwise by multiplying the points a, b and c by i, as follows. Note that in Matlab, conveniently, you can use arithmetic operations, such as multiplication, with lists of numbers instead of individual numbers. Try the following command:

```
plot([a, b, c] * i)
```

If everything works correctly, the new graph looks like the previous graph rotated 90 degrees anti-clockwise. Note that the horizontal and the vertical axes in Matlab plots might be not to the same scale, so you may wish to resize the plot window to get a better view.

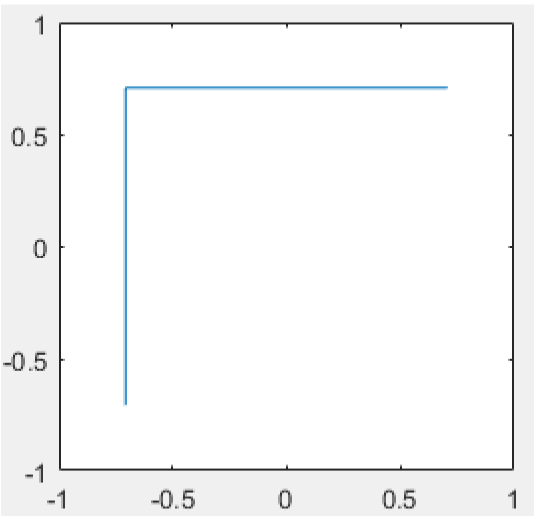
Activity 4

Now try to guess by what complex number you need to multiply the list [a, b, c] to rotate the original figure by 90 degrees clockwise, as on the following graph.



Activity 5

Now try to guess by what complex number you need to multiply the list [a, b, c] to rotate the original figure by 45 degrees anti-clockwise, as on the following graph. (Note that the complex number you use should be at the distance 1 from the origin; otherwise, the figure will be not only rotated, but also stretched or squeezed. Experiment and see.)



Activity 6

As a final exercise for today, try to guess what graph the following command produces. After you have made your guess, run this command and see if you were right.

```
plot([i, i*i, i*i*i, i*i*i*i, i*i*i*i*i])
```