Mathematics 101 Problem Set 1

1(a) Show from First principles that any constant function $f(x) = C$ satisfies $f'(x) = 0$ (meaning that $f'(x)$ is the *zero function*).

(b) Use (a) to find necessary and sufficient conditions on the coefficients a, b, c, and d to ensure that

$$f(x) = \frac{ax+b}{cx+d}$$

is a constant function.

(c) By polynomial division show that for $f(x)$ as in (b) we may write:

$$f(x) = \frac{a}{c} + \frac{bc-ad}{c(cx+d)}$$

and hence draw the same conclusion as in (b). What is

$$\lim_{x\to\infty} \frac{ax+b}{cx+d}?$$

(d) Find the equation of the inverse function $f^{-1}(x)$, where $f(x)$ is as in (b).

2(a) Find a function $f(x)$ such that:

$$f(2x+3) = x^2 + 1.$$

Hint: we have an equation of the form $f(g(x)) = h(x)$ and we want $f(x)$, so replace x by $g^{-1}(x)$.

(b) Find a linear function $f(x) = ax + b$ such that $f(f(x)) = 2x + 1$.

3. Find from First Principles the derivatives of the functions with the following rules:

(a) $2x + 1$; (b) $1 - x^2$; (c) $\frac{1}{1+x}$.

4. Find the equations of the two tangents to the curve $y = x^2$ that pass through the point $(2, 0)$.

5. Find the derivative of the cosine function from First Principles.

6. Prove that $f(x) = cx + d$ $(c, d \in \mathbb{R},\ c \neq 0)$ is continuous by taking $\delta = \frac{\epsilon}{|c|}$ in the definition of continuity.

7. Solve the following inequalities:

(i) $\frac{x-1}{1-3x} > 7$; ii) $|3x - 4| \leq 20$; (iii) $|-2(1 - 5x)| > 8$.

8. Given that $|x - 2| \leq 3$ and $|y + 2| < 1$, what is the set of all possible values of $2x + 3y$?

9. Find all values of x such that

$$|x| + |-5x| = 10.$$

10. Solve

$$|2x + 1| = 1 + |1 - 3x|.$$